Optimization of Installation of FACTS Device in Power System Planning by both Tabu Search and Nonlinear Programming Methods

Yutaka Matsuo and Akihiko Yokoyama
Department of Electrical Engineering, University of Tokyo
7-3-1, Hongo, Bunkyo-ku, Tokyo, Japan
matsuo@syl.t.u-tokyo.ac.jp, yokoyama@syl.t.u-tokyo.ac.jp

Abstract: FACTS devices such as phase shifters and series capacitors enable us to control active power flow and to avoid thermal constraints on transmission lines, resulting in an increase of the network loadability and a reduction of production cost. However where to place these devices is an essential matter because their effects considerably depend on the location. From an economic point of view, the investment cost must be taken into account as well as the reduction of the production cost. Therefore we maximize return on investment(ROI) by searching not only the best location of FACTS devices but also the rating of each device. This problem is expressed as combinatorial optimization problem nested by nonlinear optimization problem. To solve this problem, tabu search incorporated with nonlinear programming method is used. Numerical results are shown for a 41-line test system.

Keywords: FACTS device, OPF, phase shifter, series capacitor, tabu search, nonlinear programming

I. INTRODUCTION

Nowadays, it is becoming more and more difficult to obtain the right of way for building new transmission lines. Therefore FACTS devices, which can control the power flows and improve the transmission capacity of the current transmission network, has been considered to be placed into the network. Especially in a meshed network, congestion problem of active power flow becomes so critical that the active power flow control is required in order to improve the transmission capacity.

In this paper, we focus on two kinds of FACTS devices, such as phase shifter(PS) and series capacitor(SC) for the above purpose. Installing a phase shifter can make the flow on the bottleneck line pass through other paths, resulting in the increase of transmission capacity. While installing a series capacitor can change reactance of the line to reduce the overloaded flow. Using these devices we can increase power production with cheaper cost and decrease the total production cost[1][2]. On the other hand, we must take into account investment cost to achieve the above-mentioned reduction of the production cost. From an economic point of view, return on investment(ROI) is an important figure. For power system planners, it can be of great use to seek the investment plan with large ROI[3].

Here the aim of our research is to find how to place FACTS devices, that is to say the number, the location and the rating of multiple phase shifters and/or series capacitors that give the maximal ROI. It is important to find a sub-optimal solution in a practical case since this problem consists of combinatorial optimization problem.

II. OPTIMIZATION PROBLEM

In this optimization problem, DC model is used for power flow, and (N-1) rule is considered for security. In DC model a phase shifter can be treated as a voltage source inserted to the branch. We also regard a series capacitor as a voltage source inserted to the branch. The voltage divided by the current (i.e. the active power in AC model) on the branch indicates the equivalent resister value of the series capacitor(i.e. the reactance of the series capacitor). Since the current is different in each (N-1) case, the variable for the voltage in each (N-1) case must be prepared subject to equal constraint of the reactances of the series capacitors in all (N-1) cases.

[objective function]

$$\text{Max. ROI} = \frac{\text{Return}}{\text{Investment}} = \frac{C_0 - C}{\text{Investment}} \qquad (1)$$

where

$$\begin{aligned}\text{Investment } &= \sum_{i\in B} u_{ij}\left(I1 + \left(I2 + I3\alpha_{ij}^{\max}\right) f_{ij}^{\max}\right) \\ &+ \sum_{i\in B} v_{ij}\left(I4 + I5\ \gamma_{ij}^{\max}\left(f_{ij}^{\max}\right)^2\right)\end{aligned}$$

$$\text{C} = \sum_{i\in ND_G} cost_i P_i + \sum_{i\in ND_L} pcost_i L_i^{\text{penalty}}$$

[suject to]

(Kirchhoff's current law)

$$\textstyle\sum_j f_{ij} = P_i - L_i \ \ (i \in N)$$

(Kirchhoff's voltage law)

$$\delta_j - \delta_i = \alpha_{ij} + \gamma_{ij} - x_{ij} f_{ij} \ \ (ij \in B)$$

(production constraint)

$$P_i^{\min} \le P_i \le P_i^{\max} \ \ (i \in ND_G)$$

(PS angle constraint)

$$-\alpha_{ij}^{\max} \le \alpha_{ij} \le \alpha_{ij}^{\max} \ \ (ij \in B)$$

(SC capacity constraint)

$$-\gamma_{ij}^{\max} \le \frac{\gamma_{ij}}{f_{ij}} \le \gamma_{ij}^{\max} \ \ (ij \in B)$$

Series capacitors can be operated both in capacitor mode and in reactor mode.

(thermal constraint)

$$-f_{ij}^{\max} \le f_{ij} \le f_{ij}^{\max} \ \ (ij \in B)$$

(penalty cost of outage loads)

$$L_i^{\text{penalty}} \ge L_i^{\text{demand}} - L_i, \quad L^{\text{penalty}} \ge 0 \ \ (i \in ND_L)$$

(constraint related to PS and SC setting)

$$\begin{gathered} u_{ij} = 0 \ or \ 1, \ \ v_{ij} = 0 \ or \ 1 \\ \textstyle\sum_{ij\in B} u_{ij} + \sum_{ij\in B} v_{ij} \ge 1 \\ 0 \le \alpha_{ij}^{\max} \le u_{ij}\alpha_{ij}^{\max} \\ 0 \le \gamma_{ij}^{\max} \le v_{ij}\gamma_{ij}^{\max} \ \ (ij \in B) \end{gathered}$$

((N-1)rule)

In case 1,...,case N where one transmission line is tripped off, (Kirchhoff's current law), (Kirchhoff's voltage law) and (thermal constraint) are imposed. And the capacity of a series capacitor must be equal in all cases.

(SC capacity equal constraint)

$$\left.\frac{\gamma_{ij}}{f_{ij}}\right|_{\text{case 1}} = \left.\frac{\gamma_{ij}}{f_{ij}}\right|_{\text{case 2}} = \ldots = \left.\frac{\gamma_{ij}}{f_{ij}}\right|_{\text{case N}}$$

where C_0,C: the production cost before and after placing FACTS devices

ND: the set of all nodes, B: the set of all branches

ND_G, ND_L: the subset of all generator nodes, load nodes

u_{ij}: 0-1 variable that represents whether the phase shifter on branch ij is set or unset

v_{ij}: 0-1 variable that represents whether the series capacitor on branch ij is set or unset

$cost_i$: the cost per unit production of generator i

$pcost_i$: the penalty cost per unit interrupted active power of load i

P_i: the active power production of generator i ($P_i = 0$, when $i \notin ND_G$)

$P_i^{\min}, P_i^{\max}$: minimal/maximal production of generator i

L_i: the active power supplied to load i ($L_i = 0$ when $i \notin ND_L$)

L_i^{demand},L_i^{penalty}: the active power demanded by load i, the interrupted active power

$\alpha_{ij}, \alpha_{ij}^{\max}$: the phase shifter angle/maximal angle on branch ij

γ_{ij}: the equivalent voltage of the series capacitor on branch ij

$\gamma_{ij}^{\max}$: the maximal compensation reactance of the series capacitor on branch ij

f_{ij}: the active power on branch ij from i to j

$f_{ij}^{\max}$: the maximal admissible active power flow on branch ij

δ_i: the voltage phase of node i

x_{ij}: the reactance of branch ij

$I1, I2, I3, I4, I5$: constants

It should be noted that maximization of ROI leads to minimization of C because the variables such as productions of generators, the phase shifter angle and the series capacitor capacity are independent of the facility variables such as FACTS device set/unset, the maximal phase shifter angle and the maximal capacity of series capacitor. Therefore C represents the production cost for the economic load dispatch.

III. OPTIMIZATION OF RATING OF FACTS DEVICE

This problem consists of the combinatorial optimization problem that seeks where to place FACTS devices nested by nonlinear optimization problem that seeks the rating of each FACTS device. To begin with, we refer to optimization of the rating of the FACTS devices when the location of each device is settled. If the location is given, the problem turns to be a constrained nonlinear optimization. To solve this problem, we use SQP (Successive Quadratic Programming) method, that is said to be an efficient method to solve a general constrained nonlinear optimization problem. As related chap.IV, to determine the location of each device we employ tabu search, by which a sub-optimal solution can be reached in a given iteration number. Therefore to reduce computing time of SQP method is essential for practical use.

A. Limitation of (N-1) Rule

In a transmission network it is required to satisfy thermal constraints of all branches in all (N-1) cases. Though the total number of thermal constraints would be very large, the constraints necessary to get consistent solutions is a small part of all thermal constraints, that concerns the weakest part of the network in some specific (N-1) cases. Even if the other constraints are ignored, the solution to be obtained doesn't change. Therefore the proper limitation of the thermal constraints seems to play an important role to reduce the computing time.

Here a dynamic limitation of (N-1) cases and branches under consideration is needed. The following algorithm is proposed:

Denote $t_i (i = 1, 2, ..., N)$ as each (N-1) case and T as the set of t_i. Denote s_i^j as the thermal constraints of branch j in (N-1) case t_i and S_i as the set of s_i^j.

1. Give T^{init} and S_i^{init}, that is the initial set of T and S_i. Set $T = T^{init}$ and $S_i = S_i^{init}$.
2. Solve the constrained nonlinear optimization problem considering only (N-1) cases of $t_i \in T$ and thermal constraints of branch $s_i^j \in S_i (i = 1, \dots, N)$ by SQP method.
3. If the obtained solution satisfies all the constraints including constraints which are not considered at the previous step 2, accept it.
4. Otherwise remove the $s_i^j \in S_i$ which is not active from S_i. If S_i turns to be ϕ, remove t_i from T.
5. Among the set of the violated constraints not included in S_i, choose the most greatly violated constraints s_i^j and add it to S_i. If S_i turns not to be ϕ, add t_i to T.
6. If violated constraints don't exist anymore or the element number of S is more than a given number, go to 2. Otherwise go to 5.

Though this algorithm seems not to be efficient due to necessity to apply SQP method repeatedly, the number of (N-1) cases or branches under consideration decrease so drastically that the computing time would be short enough to apply this algorithm. In this algorithm the optional procedure to cope with the cycling must be implemented.

B. *Local Optimum*

This nonlinear function has no local optimum in case of considering only phase shifters but has local optimum theoretically in some cases considering series capacitors. However because an initial point rarely affects the obtained solution as shown in chap.V, we regard the first visited local optimum as a global optimum. In a larger network, it is necessary to avoid being struck into the local optimum.

IV. SEARCH FOR THE LOCATION OF FACTS DEVICE

The problem to seek the location of FACTS devices is a combinatorial optimization problem. Obviously FACTS devices must be placed to reduce the flow on the initial bottleneck, that may make a new bottleneck. Another FACTS device to cope with the new bottleneck may make another bottleneck. The locations of the FACTS devices by which the bottlenecks can be eliminated are to some degree bounded near the bottlenecks. The good location of FACTS devices would be alike to the other good solutions and the neighbor of the good location (e.g. adding, eliminating or moving the FACTS device) can be also a good one.

For this reason the neighborhood search which proceeds from current point to the neighbor is efficient. But a simple way such as greedy neighborhood search would cause the problem of being struck in a local optimum. Another problem is that the only one solution can be obtained by the method. Our model employs some approximations and the optimal solution in our model is not always optimal in practice. It is better to show some sub-optimal solutions.

Here we employ tabu search, that is based on neighborhood search and has a mechanism to escape from the local optimum.

A. *Tabu Search*

Tabu search is one of modern heuristic search methods for combinatorial optimization problems, based on neighborhood search with local optima avoidance, which models human memory processes. Using a tabu list, that retains attributes of moves of the solutions, specific moves are prohibited for a certain number of iterations to avoid trapping into a local minimum. This helps to avoid cycling, and serves also to promote a diversified search of the solutions[4].

In our problem, the location of FACTS device is expressed as a bit string that has the length equal to the number of all lines and implies that a FACTS device on each branch is set or unset. Two bit strings are needed for phase shifters and series capacitors. The neighbor of the current point is defined as addition or elimination of one phase shifter or series capacitor, that is the reverse of one bit. And the reversal of the current move is defined as tabu.

In each iteration, after the location is determined by the bit strings, ROI is obtained by optimization of rating of each device using SQP method.

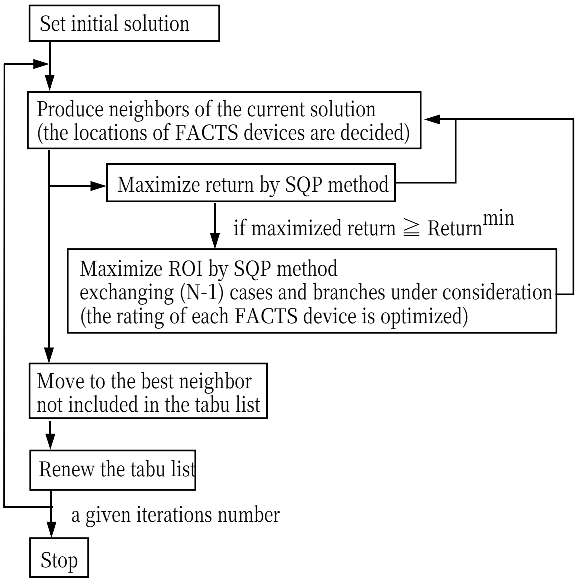


Fig. 1: Flowchart

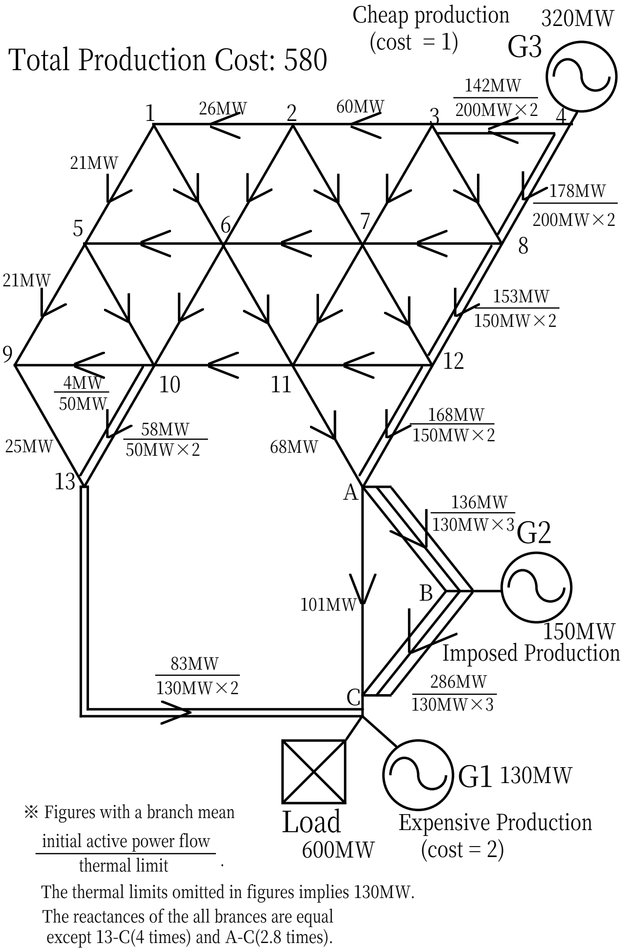
B. *Lower Boundary of Return*

In our problem, ROI is in general larger when we set fewer FACTS devices than when we set a large number of FACTS devices. In practice, however, solutions which produce greater returns and not the largest ROI are often desirable. Therefore we settle the lower limit of return and maximize the ROI as long as the return is greater than a given value, $\text{Return}^{\text{min}}$. The following constraint is added to the problem in chap.II.

$$C_0 - C \geq \text{Return}^{\text{min}} \qquad (2)$$

However it sometimes happens that the location of FACTS device determined by the bit strings can't meet the constraint even if the rating of the FACTS device is properly settled. In this case the search point should move toward where the above constraint will be satisfied. Therefore after the location is determined by the bit strings, we first maximize the return by SQP method and if the above constraints are satisfied, we maximize ROI, that becomes an objective function value. Otherwise return divided by a large constant turns to be an objective function value. This makes the search point shift to make the return greater until the above constraint is satisfied and after that is satisfied, ROI is maximized. The whole algorithm is shown in Fig. 1.

V. NUMERICAL RESULTS



※ Figures with a branch mean $\frac{\text{initial active power flow}}{\text{thermal limit}}$.
The thermal limits omitted in figures implies 130MW.
The reactances of the all brances are equal
except 13-C(4 times) and A-C(2.8 times).

Fig. 2: Initial load flow condition in model system

The model power system is shown in Fig. 2. Tabu length is set to 3, upper limit of the maximal angle of a phase shifter is set to 20° and the upper limit of the compensation reactance of a series capacitor is set to be 2.0 p.u. which is equal to the reactance of branches except 13-C and A-C. The constant in the objective function is given as I1=I4=20500, I2=12.8, I3=4.7 and I5=0.4.

The lower limit of return is fixed as 0, 20 or 40. Results are shown in Tables 1,2 and 3. Since only one load pattern is used in this study, the absolute value of the operating angle of a phase shifter is equal to the maximal angle of the phase shifter, so is a series capacitor. Therefore only the operating angle/capacity is written in the tables. Return and investment value is not the actual one, therefore the ROI doesn't have the practical meaning of how much year to take to collect the investment cost. In these tables, the branch with multiple lines such as A-B means placing the FACTS device on one of the multiple lines.

For lack of space, only the case of Table 1 is discussed below: Computing time was around 19 minutes on Sun Ultra1 compatible with Ultra SPARC 200MHz CPU. The number of iteration in tabu search was 18

Table 1: The best 5 solutions ($\text{Return}^{\min} = 0$)

ROI	FACTS status	Cost(Return)	Inv.
0.1410	-5.27° PS on A-B	562.00(17.89)	25384
0.1132	$-0.236pu$ SC on A-C	567.38(12.51)	22094
0.1077	1.76° PS on A-C	567.38(12.51)	23237
0.0961	8.97°PS on 9-13 -5.27°PS on A-B	554.42(25.47)	53026
0.0957	8.26°PS on 9-13 11.88° PS on 13-C $-0.236pu$ SC on A-C	542.21(37.69)	78731

Table 2: The best 5 solutions ($\text{Return}^{\min} = 20$)

ROI	FACTS status	Cost(Return)	Inv.
0.0961	8.97°PS on 9-13 -5.27° PS on A-B	554.42(25.47)	53027
0.0957	8.26° PS on 9-13 11.88° PS on 13-C $-0.236pu$ SC on A-C	542.21(37.69)	78731
0.0947	8.26° PS on 9-13 8.48° PS on 13-C -5.27° PS on A-B	542.04(37.85)	79941
0.0947	8.26° PS on 9-13 11.99° PS on 13-C 1.76° PS on A-C	542.04(37.85)	79941
0.0938	8.26° PS on 9-13 13.00° PS on 13-C	553.00(26.89)	57321

because the cycling problem occurred. Totally 702 locations of FACTS devices were tested, that is quite small because the total number of locations of two or one FACTS device is about 1700 and that of three or less is about 70000 in this 41-line network. The obtained solutions from 1st best to 4th best are validated to be the same as the best 4 obtained in the all search space to place two or one FACTS devices. This network has 41 (N-1) cases and about 1700 thermal constraints. By limitation of (N-1) cases, the average number of (N-1) cases and thermal constraints under consideration was respectively 6.1 and 6.4 for one iteration.

In the initial state of the network, the bottleneck line is the energized lines of branch B-C with one line open of branch B-C. And FACTS devices make the flow on branch A-B,B-C go through branch A-C, resulting in the increase of transmission capacity. Consequently branch A-C turns to be a bottleneck, and more increase of transmission capacity through the left lower corridor, that is branch 13-C, is required. As a result the flow toward branch 13-C causes a new bottleneck on branch 9-10 with branch 5-9 open or branch 10-13 with branch 9-13 open. Therefore it is necessary to place FACTS devices around node 13 for the best use of branch 13-C.

To investigate the local optimum in SQP method, we changed the initial solution of SQP method but the best 5 solutions in Table 1 don't change.

Table 3: The best 5 solutions ($\text{Return}^{\min} = 40$)

ROI	FACTS status	Cost(Return)	Inv.
0.0771	12.79° PS on 9-13 8.11° PS on 13-C -5.27° PS on A-B $1.97pu$ SC on 9-10	539.42(40.47)	104952
0.0761	7.59° PS on 13-C -5.27° PS on A-B $1.34pu$ SC on 9-13 $1.34pu$ SC on 9-13	537.52(42.37)	111316
0.0758	10.86° PS on 5-9 8.38° PS on 9-13 6.15° PS on 13-C -5.27° PS on A-B	539.19(40.70)	107388
0.0753	10.85° PS on 5-9 5.72° PS on 13-C -5.27° PS on A-B $2.00pu$ SC on 9-13	537.00(42.89)	113860
0.0751	19.24° PS on 5-9 8.38° PS on 9-10 6.15° PS on 13-C -5.27° PS on A-B	539.19(40.70)	108333

VI. CONCLUSION

This paper has presented the optimization method based on tabu search and non-linear programming methods of the location and the rating of the FACTS devices from an economic point of view. Where to set the FACTS devices and how much investment should be paid will become an important matter because the effect of the FACTS devices would change drastically due to the location. Our further work is to investigate for a larger-scale network and multiple load conditions.

VII. REFERENCES

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